Exercise 2.2.8

(Working backwards, from flows to equations) Given an equation $\dot{x} = f(x)$, we know how to sketch the corresponding flow on the real line. Here you are asked to solve the opposite problem: For the phase portrait shown in Figure 1, find an equation that is consistent with it. (There are an infinite number of correct answers—and wrong ones too.)

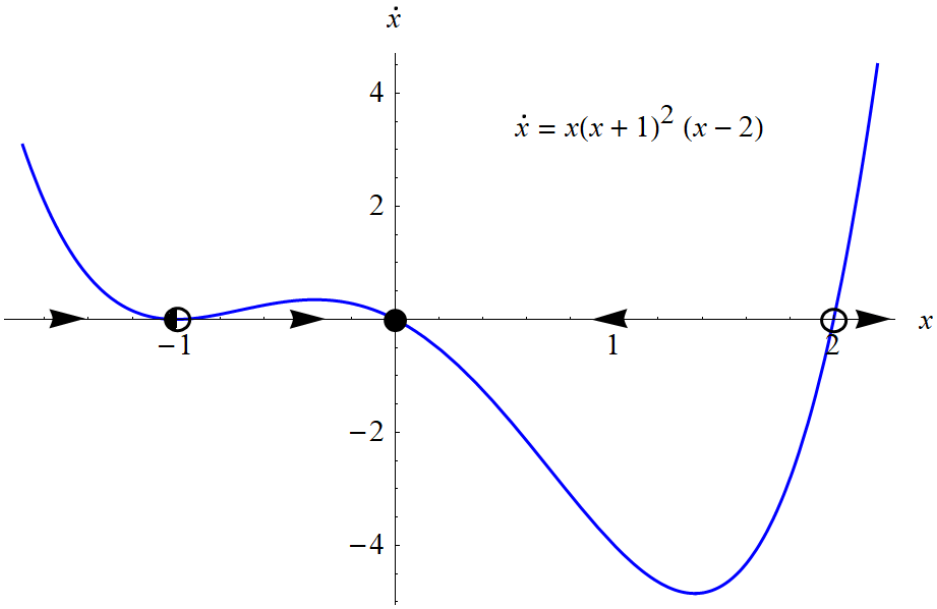
Figure 1

Solution

One possible equation is

$$\dot{x} = x(x+1)^2(x-2).$$

Plot $\dot{x}$ versus x to verify that it actually does give the desired phase portrait.



When the function is negative the flow is to the left, and when the function is positive the flow is to the right. The fixed point at $x^* = -1$ is locally half-stable, the fixed point at $x^* = 0$ is locally stable, and the fixed point at $x^* = 2$ is locally unstable.